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PHYSICS Summer Tasks

Errors and Uncertainty

In any practical subject it is impossible to be infinitely precise. You will always be limited by how accurate your equipment is and how well you have carried out your experiment. So for example, if you the very simple experiment of measuring the length of a piece of paper with a ruler, you can probably measure accurately to about the nearest half a millimetre. A quoted result of 13.2 cm is believable. A result of 13.245533352 cm is simply not justifiable with a typical ruler. It is important not to quote a result to a level of precision which you cannot justify either with the equipment you are using or by the experimental method which you employ.

Error in Equipment

As a general rule if you quote a result then there is some error implied and there will always be some error associated with using a piece of equipment. So taking the example used above, the result of 13.2 cm is assumed to be to the nearest mm as this is about as precise as we can be with a standard ruler. Therefore if the real answer were anything down to and including 13.15 cm, you would still have quoted the answer 13.2 cm. Similarly if the answer had been anything up to 13.25 cm, you would have given the answer 13.2 cm.

We could say this with the expression

$$13.15cm \leq 13.2cm < 13.25cm$$

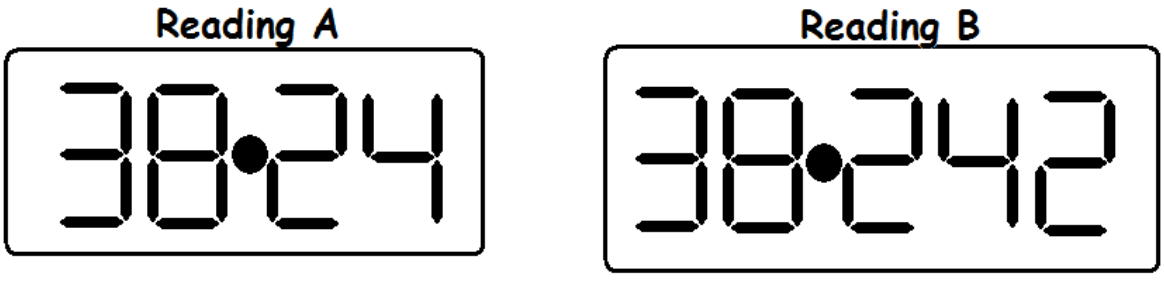
But this a convoluted way to express uncertainty in a result. A better way to show this would be to write

$$13.2cm \pm 0.05cm$$

This can also be expressed as a percentage error. As 0.05 is 0.4 % of 13.2 we could write this as:

$$13.2cm \pm 0.4\%$$

Depending on the equipment you use, there will be different errors associated. If you consider the following two measurements of mass, taken on two different digital balances:



Reading A including error would be written:

$$38.24g \pm 0.005g$$

Whereas reading B including error would be written:

$$38.242g \pm 0.0005g$$

Because the balance for reading B is 10 times more precise, the error is only one tenth of reading A. This is entirely down to the equipment used.

Experimental Error

Any experiment is going to contain some error. This tends to present itself as a spread of results for the same outcome in an experiment. It is useful to be able to attach a degree of uncertainty to your results such that you can quote error in a value. Here is how you go about doing it.

Consider an experiment where you had to measure the time something took. You used a stop watch which was accurate to 0.1s. You repeated the experiment 5 times. Here are your results.

Trail	1	2	3	4	5
Time (s)	3.9	3.5	3.7	3.4	3.5

Here is what you do:

Step 1: find the average

$$Average = \frac{3.9 + 3.5 + 3.7 + 3.4 + 3.5}{5} = 3.6s$$

Step 2: find the difference between each result and the average:

Trail	1	2	3	4	5
Time (s)	3.9	3.5	3.7	3.4	3.5
Result – average (s)	0.3	-0.1	0.2	-0.2	-0.1

Step 3 Square all the differences:

Trail	1	2	3	4	5
Time (s)	3.9	3.5	3.7	3.4	3.5
Result – average	0.3	-0.1	0.2	-0.2	-0.1
(Result – average)^2 (s^2)	0.09	0.01	0.04	0.04	0.01

Step 4. Average these squares differences:

$$aveage\ of\ square\ differences = \frac{0.09 + 0.01 + 0.04 + +0.04 + 0.01}{5} = 0.038$$

Step 5. Square root the answer:

$$final\ error = \sqrt{0.038} = 0.19s$$

So the final answer is.....

3.6s ± 0.2s (or 3.6 ± 6%)

This is also known as the standard deviation.

Those steps again summed up:

1) Find the average;
2) Find the difference between the average and the original answers.
3) Square the differences.

4) Average the squares.
5) Square root the answer.

Combining Errors

There will be occasions where you are required to find the error based on a calculation. Let us say that you have two quantities, we will call them A and B. Each one has a small error associated with it. We will use some examples for A and B. So let us say that these two quantities are

$$A = 65cm \pm 3cm$$

And

$$B = 28cm \pm 2cm$$

We can also express a and b as percentage errors of A and B which we will write down as:

$$A = 65 \pm 5\%$$

And

$$B = 28 \pm 7\%$$

Error in Adding and Subtracting

If you wanted to find the total error in adding or subtracting A and B you need to add the errors together separately. You do this with the real numbers – not the percentages:

$$A + B = 93cm \pm 5cm$$

And

$$A - B = 37cm \pm 5cm$$

Error in multiplying and dividing

If you wanted to find the total error in multiplying or dividing A and B you need to add the errors together separately but this time you use the percentage errors.

$$AB = 1820cm \pm 12\%$$

And

$$\frac{A}{B} = 2.32cm \pm 12\%$$

Using Indices

If you have to use an index the result is like this:

$$A^n = 65^n \pm n \times 5\%$$

Or

$$B^n = 28^n \pm n \times 7\%$$

Errors Worksheet

QUESTION 1

Look at the following lengths:

A = 3.5m $\pm$ 0.04m

B = 6.2m $\pm$ 0.08m

C = 2.5m $\pm$ 0.1 m

D = 5.3m $\pm$ 0.09m

Using this information find the answer to the following sums including the total error:

a) A + B
b) B – C
c) AD
d) D / C
e) A^3
f) B(A + D)

Question 2

Two events, X and Y were timed using the same stop watch. Each was timed 6 times. The results are shown below:

Attempt	1	2	3	4	5	6
Time for X (s)	4.5	3.9	5.2	4.6	5.1	4.8
Time for Y (s)	9.4	10.1	9.3	8.9	9.9	9.6

Find the average value of X and Y quoting the uncertainty in each value.

Question 3

The kinetic energy which a moving object is known to be determined by the equation:

$$E = \frac{1}{2}mv^2$$

Where E = Kinetic energy of the object (J)
m = mass of object (kg)
v = velocity of object (ms^{-1})

A car is tested to find out how much energy it has at when its speedometer reads 50mph (22.2 ms^{-1}). However the car's speedometer is not very accurate. The car makes 5 attempts to move at 50 mph and its speed is measured much more accurately with some better equipment. Here are the results.

Attempt	1	2	3	4	5
Speed (ms^{-1})	21.5	23.4	22.6	20.8	21.3

The car has a mass of 1200 kg $\pm$ 25 kg.

FIND OUT THE ENERGY OF THE CAR WHEN MOVING AT 50 MPH ACCORDING TO THE SPEEDOMETER QUOTING THE UNCERTAINTY IN THE RESULT.